$$f(x) = x^2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$= \frac{2xh+h^2}{h}$$

$$= \frac{2xh}{h} + \frac{h^2}{h}$$

$$= 2x + h$$

THUS,

$$\frac{f(x+h)-f(x)}{h} = 2x+h = \text{"SLOPE OF SECANT FROM } x \text{ TO } x+h\text{"}$$

FOR EXAMPLE, IF WE WANTED THE SLOPE OF THE SECANT FROM 3 TO 10 THEN WE WOULD USE $x = 3$ AND $h = 7$ (SO $x + h = 10$) TO GET SLOPE $= 2(3) + 7 = 13$

WE WANT TO KNOW THE SLOPE AS $h \to 0$ TO GET

$$f'(x) = 2x = \text{"SLOPE OF THE TANGENT LINE AT } x\text{"}$$

# NOTES/COMMENTS

WE JUST FOUND

IF $y = f(x) = x^2$ $\Leftarrow$ HEIGHT ON GRAPH AT $x$

THEN $y' = f'(x) = 2x$. $\Leftarrow$ SLOPE OF TANGENT AT $x$

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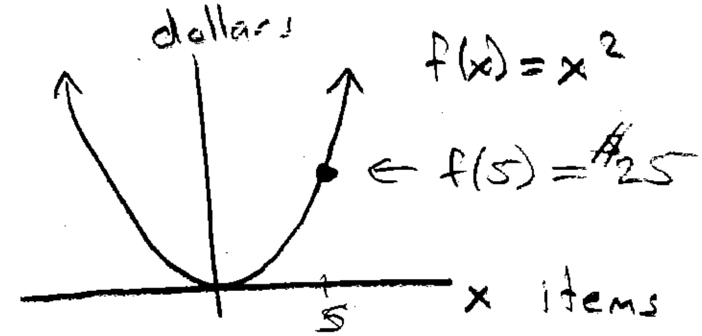
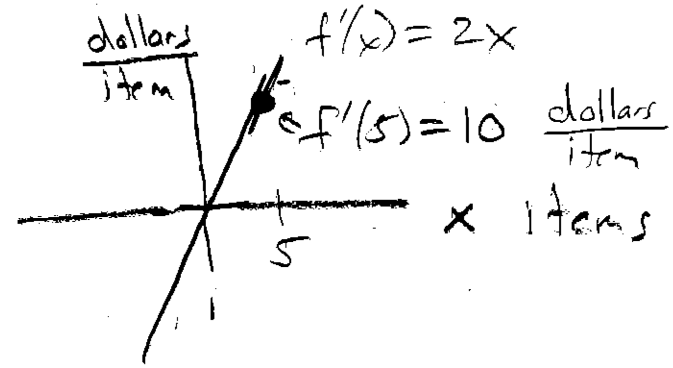



ASSUME $f(x) = x^2$ dollars in revenue

WHERE $x$ is in items

THEN $f'(x) = 2x$ is $\dfrac{\text{dollars}}{\text{item}}$.

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FOR EXAMPLE, IF YOU SELL $x = 5$ ITEMS

THEN $f(5) = (5)^2 = 25$ dollars in revenue

AND $f'(5) = 2(5) = 10$ $\dfrac{\text{dollars}}{\text{item}}$

Let's try it again:

$$f(x) = x^3$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$$

$$(x+h)^3 = (x+h)(x+h)(x+h)$$
$$= (x^2+2xh+h^2)(x+h)$$
$$= x^3+2x^2h+xh^2+x^2h+2xh^2+h^3$$
$$= x^3+3x^2h+3xh^2+h^3$$

$$= \frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$$

$$= \frac{3x^2h+3xh^2+h^3}{h}$$

$$= 3x^2+3xh+h^2$$

LETTING $h \to 0$

GIVES $\boxed{f'(x) = 3x^2}$

$\boxed{\text{ASIDE}}$ PASCAL'S TRIANGLE

$$(x+h)^2 = \overset{1}{x^2} + \overset{2}{2xh} + \overset{1}{h^2}$$

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \end{array}$$

$$(x+h)^3 = \underset{1}{x^3} + \underset{3}{3x^2h} + \underset{3}{3xh^2} + \underset{1}{h^3}$$

$$(x+h)^4 = x^4+4x^3h+6x^2h^2+4xh^3+h^4$$

etc....

SIMILARLY, $f(x) = x^4$

$$\frac{(x+h)^4 - x^4}{h} = \frac{\cancel{x^4}+4x^3h+\text{"STUFF"}-\cancel{x^4}}{h} = 4x^3 + \text{"STUFF"}$$

EACH TERM HAS AN $h$ IN IT

SO $f'(x) = 4x^3$

AND SO ON....

**POWER RULE**: If $f(x) = x^n$, then $f'(x) = n\,x^{n-1}$.

Written briefly, $\frac{d}{dx}(x^n) = n\,x^{n-1}$.

Special Cases:

$$\frac{d}{dx}(x) = 1.$$

$$\frac{d}{dx}(1) = 0.$$

*Note*: Although we won't prove this.
The power rules works for ALL powers
(including negative and decimal powers)

Examples

$\frac{d}{dx}(x^2) = 2x^1$

$\frac{d}{dx}(x^3) = 3x^2$

$\frac{d}{dx}(x^4) = 4x^3$

$\frac{d}{dx}(x^5) = 5x^4$

$\frac{d}{dx}(x^6) = 6x^5$

etc.

$\frac{d}{dx}(x^{-1}) = -x^{-2}$

$\frac{d}{dx}(x^{-2}) = -2x^{-3}$

$\frac{d}{dx}(x^{-3}) = -3x^{-4}$

$\frac{d}{dx}(x^{2.5}) = 2.5x^{1.5}$

$\frac{d}{dx}(x^1) = 1x^0 = 1$

$\frac{d}{dx}(1) = \frac{d}{dx}(x^0) = 0$

# Exponents Review

EXAMPLES!

$$\frac{1}{x^b} = x^{-b}.$$

$$f(x) = \frac{1}{x^2} = x^{-2} \Rightarrow f'(x) = -2x^{-3}$$

$$f(x) = \frac{1}{x^{1.2}} = x^{-1.2} \Rightarrow f'(x) = -1.2\,x^{-2.2}$$

$$f(x) = \frac{1}{x^{-3}} = x^{3} \Rightarrow f'(x) = 3x^{2}$$

$\sqrt[a]{x} = x^{1/a}$ and $\sqrt{x} = x^{1/2}$.

$$f(x) = \sqrt{x} = x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2}$$

$$f(x) = \sqrt[3]{x} = x^{1/3} \Rightarrow f'(x) = \frac{1}{3}x^{-2/3}$$

$$f(x) = \sqrt[5]{x} = x^{1/5} \Rightarrow f'(x) = \frac{1}{5}x^{-4/5}$$

$$x^a x^b = x^{a+b}.$$

$$\frac{x^a}{x^b} = x^{a-b}.$$

$$(x^a)^b = x^{ab}.$$

$$f(x) = \frac{x^4x^2}{x} = \frac{x^6}{x} = x^5 \Rightarrow f'(x) = 5x^4$$

$$f(x) = \sqrt{x^3} = (x^3)^{1/2} = x^{3/2} \Rightarrow f'(x) = \frac{3}{2}x^{1/2}$$

$$f(x) = \sqrt[3]{x^7} = x^{7/3} \Rightarrow f'(x) = \frac{7}{3}x^{4/3}$$

**SUM/DIFFERENCE RULE:** $\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$, and
$\frac{d}{dx}(f(x)-g(x)) = f'(x)-g'(x)$.

EXAMPLES

$$\frac{d}{dx}\left(x^{10}+x^{9}\right) = 10x^{9}+9x^{8}$$

$$\frac{d}{dx}\left(\sqrt[3]{x}+\frac{1}{x^4}\right) = \frac{d}{dx}\left(x^{1/3}+x^{-4}\right) = \frac{1}{3}x^{-2/3}-4x^{-5}$$

$$\frac{d}{dx}\left(x^{11}+7+x\right) = 11x^{10}+0+1 = 11x^{10}+1$$

**COEFFICIENT RULE:** $\frac{d}{dx}\left(cf(x)\right) = cf'(x).$

Special Cases:

$$\frac{d}{dx}(c\,x) = c.$$

$$\frac{d}{dx}(c) = 0.$$

$\boxed{\text{EXAMPLES}}$

$$\frac{d}{dx}\left(4x^3\right) = 4 \cdot 3x^2 = 12x^2$$

$$\frac{d}{dx}\left(-14x^{100}\right) = -14 \cdot 100x^{99} = -1400x^{99}$$

$$\frac{d}{dx}\left(3x^{10} - 15x^3\right) = 3 \cdot 10x^9 - 15 \cdot 3x^2 = 30x^9 - 45x^2$$

$$\frac{d}{dx}\left(6x + 4\right) = 6 + 0 = 6$$

*Derivative methods so far*:

1. **Expand** into sum of terms.

2. **Rewrite** each term as: $cx^b$.

3. Bring coefficients/sum along for the ride.

4. Use power rule.

5. Simplify.

FIND THE DERIVATIVE

1. $y = 5x - 3x^2 + 1$

$$\frac{dy}{dx} = 5 - 3 \cdot 2x + 0 = \boxed{5 - 6x}$$

2. $R(q) = -0.4q^3 + \dfrac{q^2}{2} + 4.5q = -0.4q^3 + \frac{1}{2}q^2 + 4.5q$

$$R'(q) = -0.4 \cdot 3q^2 + \frac{1}{2} \cdot 2q + 4.5 = \boxed{-1.2q^2 + q + 4.5}$$

3. $y = \sqrt[3]{x} - 3x^4 + \dfrac{5}{\sqrt{x}} = x^{1/3} - 3x^4 + 5x^{-1/2}$

$$\boxed{\frac{dy}{dx} = \frac{1}{3}x^{-2/3} - 12x^3 - \frac{5}{2}x^{-3/2}}$$

4. $f(x) = x^3\left(x^5 + \dfrac{2}{x}\right) = x^3 \cdot x^5 + x^3 \cdot \dfrac{2}{x} = x^8 + 2x^2$

$$\boxed{f'(x) = 8x^7 + 4x}$$

5. $g(x) = 12\sqrt{x} - \frac{10}{x^2} + 17 = 12x^{1/2} - 10x^{-2} + 17$

$$\boxed{g'(x) = 6x^{-1/2} + 20x^{-3} + 0}$$

6. $y = \frac{x^{-2} + x^7 - 2}{\sqrt{x}} = \frac{x^{-2}}{x^{1/2}} + \frac{x^7}{x^{1/2}} - \frac{2}{x^{1/2}} = x^{-2.5} + x^{6.5} - 2x^{-1/2}$

$$\frac{dy}{dx} = \boxed{-2.5x^{-3.5} + 6.5x^{5.5} + x^{-3/2}}$$

7. $y = \sqrt{x}(x^3 + 4) = x^{1/2}x^3 + 4x^{1/2} = x^{3.5} + 4x^{1/2}$

$$\boxed{\frac{dy}{dx} = 3.5x^{2.5} + 2x^{-1/2}}$$

8. $y = \frac{x^3}{3} + \frac{5}{x^2} + 6\sqrt[3]{x^2} = \frac{1}{3}x^3 + 5x^{-2} + 6x^{2/3}$

$$\frac{dy}{dx} = x^2 - 10x^{-3} + 6 \cdot \frac{2}{3}x^{-1/3} = \boxed{x^2 - 10x^{-3} + 4x^{-1/3}}$$

9. $y = \frac{t^2 - \sqrt{t} + 2}{t^2} = \frac{t^2}{t^2} - \frac{t^{1/2}}{t^2} + \frac{2}{t^2} = 1 - t^{-1.5} + 2t^{-2}$

$$\boxed{\frac{dy}{dx} = 0 + 1.5x^{-2.5} - 4t^{-3}}$$

$\frac{2}{3} - \frac{3}{2} = \frac{4}{6} - \frac{9}{6} = -\frac{5}{6}$

10. $y = \frac{4\sqrt[3]{x^2}}{5\sqrt{x^3}} = \frac{4}{5}\frac{x^{2/3}}{x^{3/2}} = \frac{4}{5}x^{\left(\frac{2}{3} - \frac{3}{2}\right)}$

$$= \frac{4}{5}x^{-5/6}$$

$$\boxed{\frac{dy}{dx} = \frac{4}{5}\left(-\frac{5}{6}\right)x^{-\frac{11}{6}} = -\frac{2}{3}x^{-11/6}}$$